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the sum of the absolute values of all the coefficients of $f(x) - \Delta y \equiv (x - x_0)\phi(x) - \Delta y$ is

$$\Sigma = |\alpha_0| + |\alpha_1 - \alpha_0 x_0| + |\alpha_2 - \alpha_1 x_0| + \cdots$$
$$+ |\alpha_{n-1} - \alpha_{n-2}x_0| + |-\alpha_{n-1}x_0 - \Delta x\cdot\phi(x_0 + \Delta x)|. \quad (3)$$

The α's are arbitrary. Let $\alpha_i = \alpha_{i-1}x_0$, so that $\alpha_i = \alpha_0 x_0{}^i$; then

$$\Sigma = |\alpha_0| + |-\alpha_0 x_0{}^n - \Delta x\cdot\phi(x_0 + \Delta x)|.$$

The equation $f(x) - \Delta y = 0$ must, of course, become

$$\alpha_0[x^n - (x_0 + \Delta x)^n] = 0, \qquad \text{and} \qquad \Sigma = |\alpha_0|\,(1 + |x_0 + \Delta x|^n).$$

By choosing α_0 small enough, we can make Σ as small as we please.

This type of equation may not be what is desired, though it serves very well for practice in Horner's Method. In any case, the expression (3) for Σ is a useful guide to a happy medium of simplicity.

448. Proposed by W. D. CAIRNS, Oberlin College.

In the *Washington* (D. C.) *Times*, Mr. W. A. Dayton called attention some time ago to a curious repetition of digits in the decimal value of 1/115. If this decimal, which we print in the form $.00\dot{8}6956521739130\ 4347826\dot{0}$, be divided by two, the result is $.00\dot{4}3478260\ 8695652173913\dot{0}$, the fourteen-digit and eight-digit groups having been thus interchanged. A similar result, as he points out, is obtained if the original decimal value is divided by four. Mr. Dayton asks that this curiosity be explained.

SOLUTION BY H. S. BROWN, Hamilton College, N. Y.

The decimal point was misplaced in the original question. The separation of the decimal into two *groups* of digits has no bearing on the problem.

The following theorem, found in almost any algebra that has a chapter on Theory of Numbers, applies directly: "If $1/p$ be converted into a circulating decimal with $p - 1$ figures in its recurring period, p must be prime, and the recurring period being multiplied by 2, 3, $\cdots$, $(p - 1)$ will reproduce its own digits in the same order." (C. Smith's *Treatise on Algebra*, p. 502.) Proof: If

$$1/p = .\dot{a}_1 a_2 a_3 \cdots \dot{a}_{p-1},$$

we have

$$10 = a_1 p + r_1, \qquad 10r_1 = a_2 p + r_2, \qquad 10r_2 = a_3 p + r_3, \ \cdots.$$

Now p is prime to 10, for otherwise $1/p$ would not be reducible to a pure circulator; hence p is prime to $r_1, r_2, r_3, \cdots$. But as $r_1, r_2, r_3, \cdots$, are all different and there are $p - 1$ of them, they are the numbers 1, 2, 3, $\cdots$, $(p - 1)$, not of course in order. Also from the above, p is prime. Now since $r_1, r_2, r_3, \cdots$ include all numbers 1, 2, 3, $\cdots$ $(p - 1)$, it follows that when k/p is reduced to a decimal the recurring digits will be the same as before, beginning with that one for which the previous remainder was k.

In the above example, we have

$$\frac{1}{115} = \frac{1}{5 \times 23} = \frac{2}{10 \times 23}, \qquad \text{where } k = 2.$$

It is worth noting that if the decimal 1/23 of 22 places is separated into two equal groups and the sum of the digits of the first half added to the sum of the digits of the second half, the result will consist wholly of nines. Thus, $1/23 = .\dot{0}43478260869565217391\dot{3}$.

$$\begin{array}{lr}
48 = \text{sum of the first eleven digits,} & 04347826086\\
\underline{51} = \text{sum of the second eleven digits,} & \underline{95652173913}\\
99 & 99999999999
\end{array}$$

This result is general under the condition that the denominator is prime and the number of figures in the recurring period is even. We note also that twos and fives, as factors in the denominator of a fraction, have no effect on the number of recurring figures when the fraction is reduced to a decimal.

Also solved by E. E. WHITFORD, E. B. ESCOTT, ELIJAH SWIFT, NORMAN ANNING, R. D. BOHANNAN, GEORGE BLANCHARD, R. E. GAINS, and E. F. CANADAY.

449. Proposed by FRANK IRWIN, University of California.

Sum the expression

$$1+2\binom{k+1}{k}+3\binom{k+2}{k}+\cdots+(n-k+1)\binom{n}{k}.$$

Also show how to sum

$$1\cdot 2+2\cdot 3\binom{k+1}{k}+3\cdot 4\binom{k+2}{k}+\cdots+(n-k+1)(n-k+2)\binom{n}{k},$$

$$1\cdot 2\cdot 3+2\cdot 3\cdot 4\binom{k+1}{k}+3\cdot 4\cdot 5\binom{k+2}{k}+\cdots+(n-k+1)(n-k+2)(n-k+3)\binom{n}{k},$$

etc., where $\binom{l}{k}$ is used to denote the coefficient of x^k in $(1+x)^l$.

SOLUTION BY A. M. KENYON, Purdue University.

The sum of the first n binomial coefficients in any column of Pascal's triangle is given by the formula,

$$(1)\qquad \sum_{i=0}^{n-1}\binom{k+i}{k}=\binom{k+n}{k+1}=\binom{k+n}{n-1},\qquad \begin{cases}k=0,1,2,\cdots\\ n=1,2,3,\cdots\end{cases}$$

as may be verified and established by induction.

Making use of the notation,

$$x^{(n)}=\prod_{i=0}^{n-1}(x-i),\qquad x^{[n]}=\prod_{i=1}^{n}(x+i),\qquad n=1,2,3,\cdots$$

$$x^{(0)}=x^{[0]}=1,$$

we have

$$i^{(m)}\binom{k+i}{k}=i^{(m)}\binom{k+i}{i}=k^{[m]}\binom{k+i}{k+m},\qquad m=0,1,2,\cdots$$

and this in (1) gives,

$$(2)\qquad \sum_{i=0}^{n-1}i^{(m)}\binom{k+i}{k}=k^{[m]}\binom{k+n}{k+1+m},\qquad \begin{cases}k,m=0,1,2,\cdots\\ n=1,2,3,\cdots\end{cases}$$

From the relations[1] among the coefficients of the polynomial in x which result from expanding $x^{(n)}$ we find,

$$i^{[m]}=\sum_{j=0}^{m}\binom{m}{j}^2\underline{|m-j}\,i^{(j)},\qquad m=0,1,2,\cdots$$

whence on making use of (2)

$$(3)\qquad \sum_{i=0}^{n-1}i^{[m]}\binom{k+i}{k}=\sum_{i=0}^{m}\binom{m}{i}^2\underline{|m-i}\,k^{[i]}\binom{k+n}{k+1+i},\qquad \begin{cases}k,m=0,1,2,\cdots\\ n=1,2,3,\cdots\end{cases}$$

Since the problem proposed requires the sum of the first $n-k+1$ terms, we put $n-k+1$ for n,

$$(4)\qquad \sum_{i=0}^{n-k}i^{[m]}\binom{k+i}{k}=\sum_{i=0}^{m}\binom{m}{i}^2\underline{|m-i}\,k^{[i]}\binom{n+1}{k+1+i},\qquad \begin{cases}m=0,1,2,\cdots\\ n>k=0,1,2,\cdots\end{cases}$$

Setting $m=0, 1, 2, 3$, etc., in (4), we get

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[1] See, "Some Properties of Binomial Coefficients," *Indiana Academy of Science*, 1914, p. 449.